ACTIVITY – ARITHMETIC PROGRESSIONS

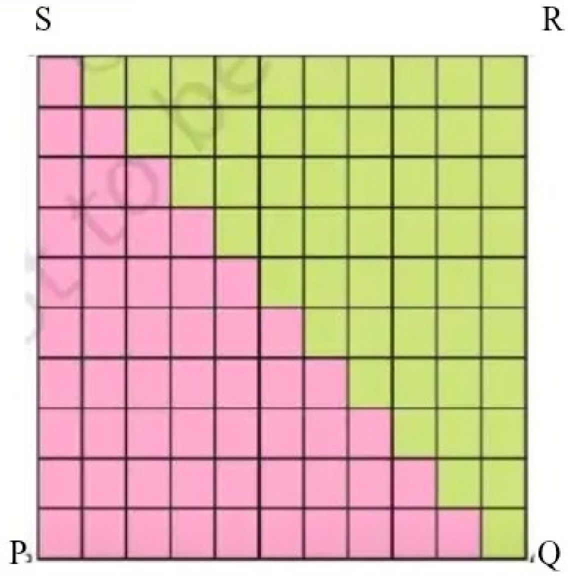
OBJECTIVE: To find the sum of first 'n' natural numbers by graphical method.

Materials Required : Graph paper , colour pencils, geometry box.

PROCEDURE: Let us consider the sum of first 'n' natural numbers

$1 + 2 + 3 + 4 + \dots\dots\dots + n$ (say n = 10)

1. Take a graph paper (14cm × 14 cm) and paste it on a white paper.
2. Mark the rectangles 1,2,3,…….n,(n+1) along the vertical line and 1,2,3,…n along the horizontal line
3. Colour the rectangular strips of length 1 cm, 2 cm, 3 cm…….n cm each of width 1 cm.
4. Complete the rectangle with sides n and n+1. Name this rectangle as PQRS.
5. Count the coloured squares and total number of squares in rectangle PQRS.



Mathematically:

Area of rectangle PQRS = 10×11

Area of shaded region = $\frac{1}{2} \times 10 \times 11 = 55$ ……………..(i)

Also, area of shaded region = $(1\times1) + (2\times1) + (3\times1) + \dots + (10\times1)$

$= 1 + 2 + 3 + \dots\dots + 10 = 55$ ………(ii)

From (i) and (ii)

$1 + 2 + 3 + \dots\dots + 10 = \frac{1}{2} \times 10 \times 11 = 55$

Hence $1 + 2 + 3 + \dots\dots + 10 = \frac{1}{2} \times 10\,(10 + 1)$

OBSERVATION:

The number of shaded squares = $\frac{1}{2} \times$ Total no. of squares

No. of shaded squares = $1 + 2 + 3 + \dots\dots\dots + n$

Total squares = Area of rectangle = $n\,(n + 1)$

$\therefore 1 + 2 + 3 + \dots + n = \frac{1}{2} n\,(n + 1)$

CONCLUSION :

Sum of first n natural numbers $= \frac{1}{2} n\,(n + 1)$